Fourier Expanation

lemma. 函数列 $\{1, \sin x, \cos x, \cdots, \sin nx, \cos nx, \cdots\}$ 正交 $x \in [-\pi, \pi]$

By if $m \neq n$, $\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \int_{-\pi}^{\pi} \sin mx \sin nx \, dx = 0$ and $\forall m, n$ $\int_{-\pi}^{\pi} \cos nx \sin mx \, dx = 0$

Fourier series. Suppose that $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ where $f(x)$'s period is 2π

then $a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos mx \, dx$, $b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx \, dx$

若周期为 $2T$: just let $\tilde{x} = \frac{\pi}{T} x$

$a_n = \frac{1}{T} \int_{-T}^{T} f(x) \cos \frac{n\pi x}{T} \, dx$

e.g. 半波整流 $\sin wt$ 去负部分 $f(t) = \begin{cases} 0 & \\ \sin wt & [0, \frac{\pi}{w}] \end{cases}$ 讨论 $n \neq 1$

$\sum \frac{1}{n^2} = \underbrace{\sum \frac{1}{(2k-1)^2}}_{\text{偶延拓}} + \underbrace{\sum \frac{1}{(2k)^2}}_{} \nearrow$ 自相似

Riemann. $\psi(x)$ $[a, b]$ 可积 or 绝对可积 分划

$\lim_{p \to +\infty} \int_a^b \psi(x) \sin px \, dx = \lim_{p \to +\infty} \int_a^b \psi(x) \cos px \, dx = 0$

$P = \frac{4}{2} \Sigma |m_i|$

Fourier Transformation

Using complex form of cos and sin

Thm. $f(x) = \frac{1}{2} \sum_{n=-\infty}^{+\infty} c_n e^{i w_n x}$ where $w_n = \frac{n\pi}{T}$ and $c_n = \frac{1}{T} \int_{-T}^{T} f(t) e^{-i w_n t} dt$, $c_{-n} = \overline{c_n}$ in Complex field

Some insight: $T \to +\infty$, $\Delta w \to 0$

$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} f(t) e^{-iwt} dt \right] e^{iwx} dw$